ME4005 Lab 3: Beam Vibration

Experimental Investigation of the Static and Dynamic Behavior of a Cantilever Beam

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Outline

- Experimental Results
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    - Strain gauge sensitivity
    - Beam Equivalent Stiffness
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Laboratory Objectives

- Perform experimentation to determine the following characteristics of a cantilever beam and strain gauge.
  - Sensitivity
  - Natural Frequencies
  - Damping Coefficient
- Manipulate the beam by adding a damping material to test impact of additional material on experimental damping coef. and natural frequencies.[1]

Experimental Setup

- A quarter bridge is set up on the side of a lab bench as seen in Figure 1 below.[1]

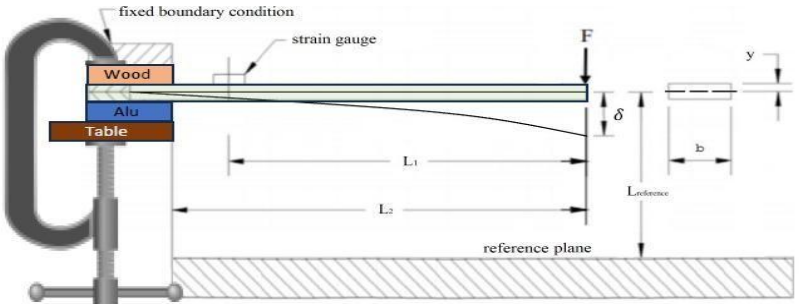


Figure 1: Experiment Set Up Instructions

- Beam tip position (initial displacement is measured via a meter stick held up to the side of the beam from the floor. The resting position is subtracted to get the displacement value. Figure 2 shows my experimental setup.

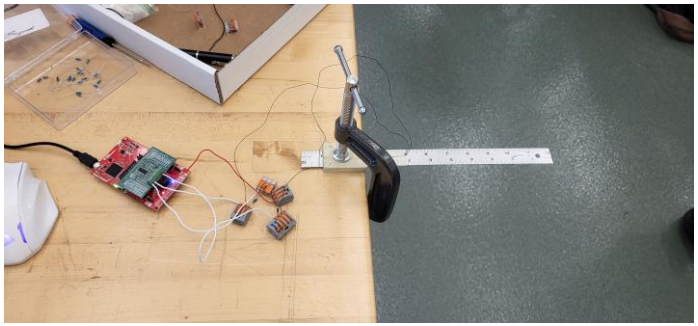


Figure 2: Experiment Set Up Image

Static Measurements: Strain Gauge Sensitivity

- Calculating the base sensitivity of the strain gauge allows for proper conversion of deformation to force or other useful units
- The tabulated data was obtained by placing coins of known mass on the end of the beam and observing the response of the system at steady state.
- The output of the avg strain gauge voltage tracks linearly with displacement.
- Sensitivity is estimated from the slope of the Voltage v Displacement Curve
- Assumptions
  - Strain gauge is perfectly mounted
  - Temperature is constant
  - Material behaves linearly [2]
- As seen in the graph the linear relationship very strong but does not perfectly align at all values (likely due to real world noise)

Table 1: Calibration Data Table

Dataset	Beam Tip Position [mm]	Beam Tip Load [N]	Average Strain gauge Voltage [mV]
1	0	0	8.41
2	3.175	0.0556	8.58
3	6.350	0.1112	8.68
4	7.938	0.1603	8.80
5	9.525	0.2047	8.90

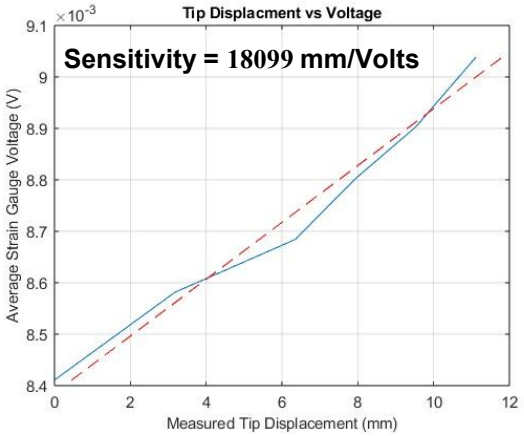


Figure 3: Tip Displacement vs Voltage Graph

Static Measurements: Beam Equivalent Stiffness

- By modeling the beam as a spring mass system, we can utilize spring mass vibration equations to solve for system values.
- The linear displacement load relationship is indicative of loading within the material's elastic deformation range
- The equivalent stiffness is the slope of load v displacement
- Assumptions
  - Beam remains within elastic deformation

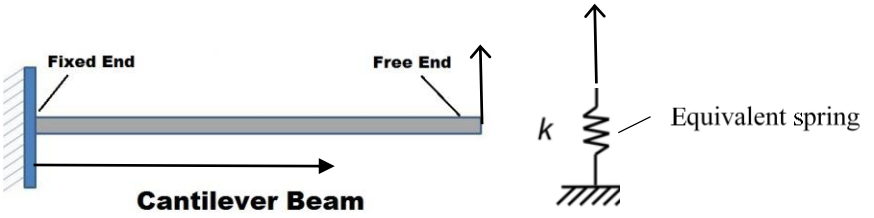


Figure 4: Cantilever Beam Diagram & Equivalent spring approximation diagram

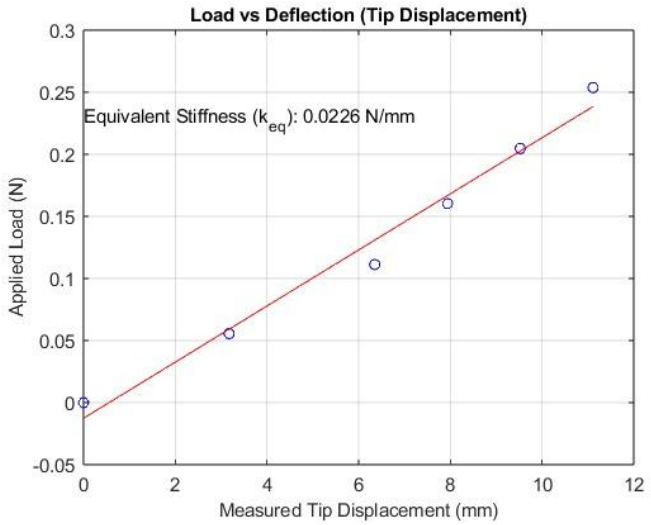


Figure 5: Load vs Tip Deflection Graph

Dynamic Measurements: Initial Displacement Excitation

- The initial displacement was provided by depressing the tip of the beam from rest at approx. 92cm above floor to 90cm above floor.
- The results show a decay of the amplitude of the displacement experienced in the beam after release from initial displacement. This can be explained by innate material dampening (structural) and air resistance (frictional).

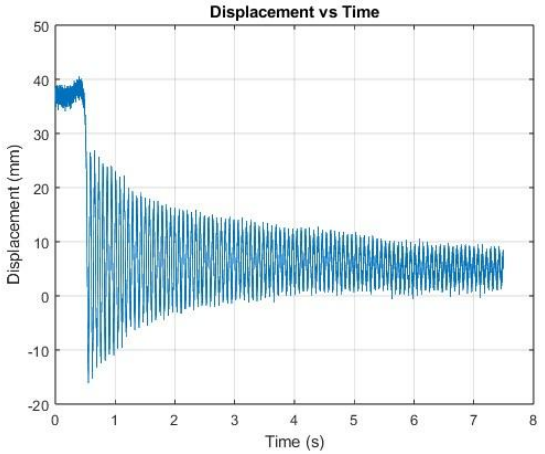


Figure 6: Displacement vs Time Graph

Dynamic Measurements: Initial Displacement Excitation

- To estimate the damping ratio, damped natural frequencies, and system natural undamped frequency the system is assumed to be modeled by the below ordinary differential equation. To calculate the damping ratio and damped natural frequency the logarithmic decrement method is used (also shown in equations below) [3]

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = 0 \qquad \delta = \ln\left(\frac{x_i}{x_{i+1}}\right) = \frac{1}{n}\ln\left(\frac{x_i}{x_{i+n}}\right) \qquad \zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

- The parameters calculated and displayed in the figure below show an underdamped system. This is expected for real world materials in free vibration as air resistance and material damping make un-damped vibration impossible.

Table 2: Initial Displacement System Values

Parameters	Value
Natural Freq. (Hz)	14.2303
Damping Ratio (%)	0.2489
Damped Nat. Freq. (Hz)	14.2292

Dynamic Measurements: Initial Displacement Excitation

- Equation below models the response

$$x(t) = w_0 e^{-\zeta \omega_n t} \cos(\omega_d t)$$

- The theoretical model seems to decay too sharply indicating that the calculated damping coefficient is higher than actual.

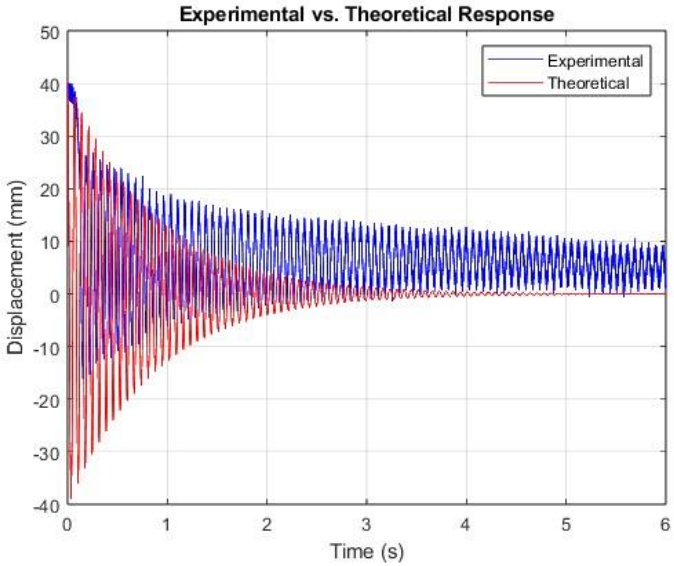


Figure 7: Experimental vs Theoretical Graph

Dynamic Measurements: Initial Displacement Excitation

- Using a 3 second observation window a FFT function was applied to the signal to obtain a frequency magnitude graph.
- As can be seen by the spectrum there are no other significant frequencies indicating that as expected the initial displacement put the beam into primarily mode one vibration.
- The fundamental frequency can be easily seen in the only positive dB peak that occurs at 14.22 Hz
- The estimated frequency appears as expected on this spectrum graph as the lone positive Hz peak

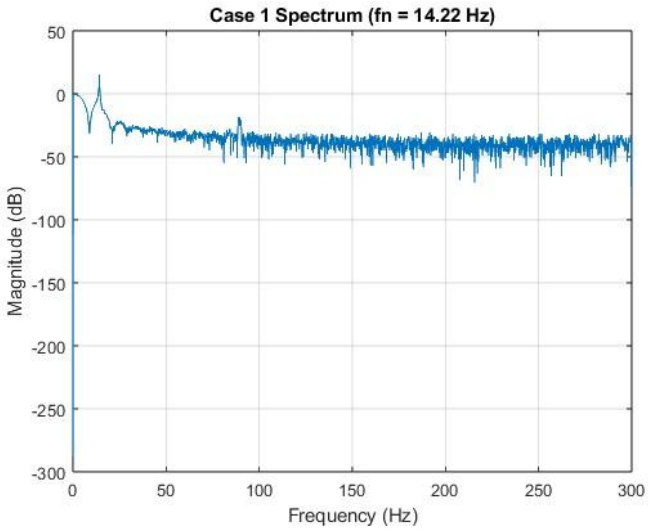


Figure 8: Case 1 Frequency Spectrum Graph

Dynamic Measurements: Impulse Excitation

- The impulse displacement trial was conducted by dropping a coin from as set height onto the beam while collecting data.
- As can be seen in the graph to the right the impulse excitation behaved very similarly to case 1 initial displacement excitation.

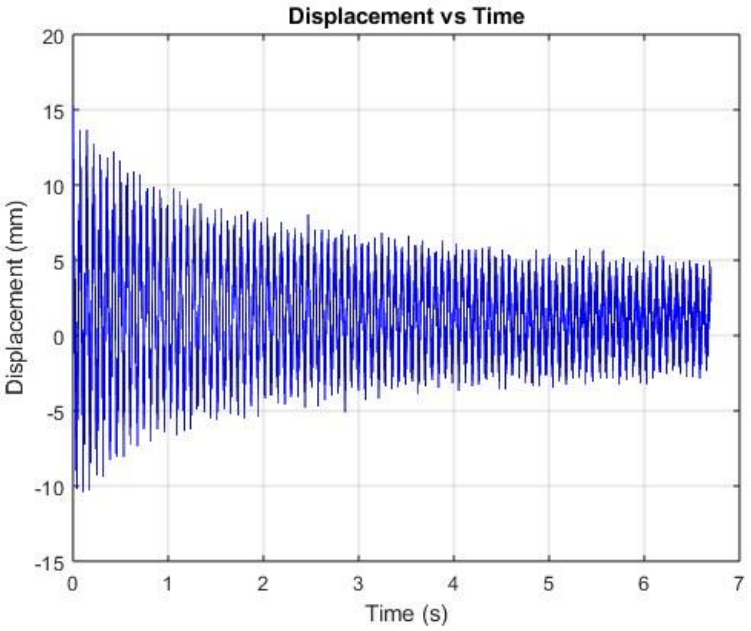


Figure 9: Case 2 Impulse Displacement: Displacement v Time Graph

Dynamic Measurements: Impulse Excitation

- Using a 3 second observation window a FFT function was applied to the signal to obtain a frequency magnitude graph.
- As can be seen by the spectrum there are other less significant mode states at approximately 75 and 245 Hz respectively.
- The fundamental frequency can be easily seen in the only positive dB peak that occurs at 14.22 Hz
- The estimated frequency appears as expected on this spectrum graph as the lone positive Hz peak

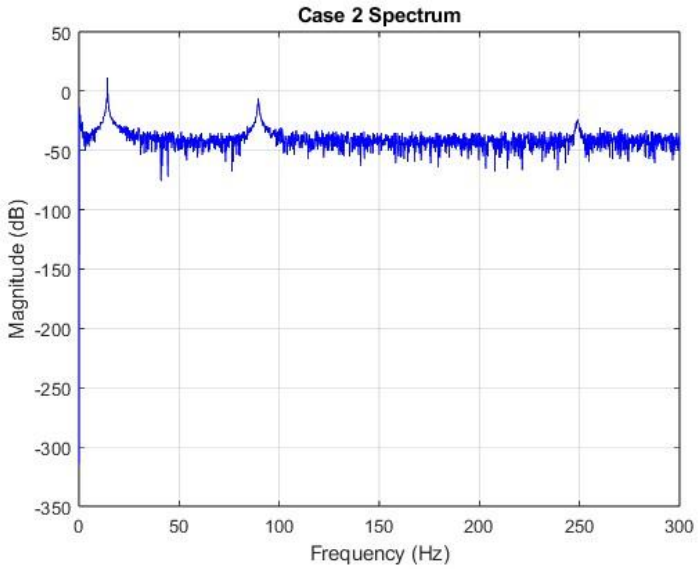


Figure 10: Case 2 Frequency Spectrum Graph

Vibration Control: Viscoelastic Damping

- A five-inch piece of tape was attached to the bottom of the beam for cases 3 and 4.
- Case 3 used carpet tape shown
- Case 4 used brand name duct tape shown partially out of frame to the right.

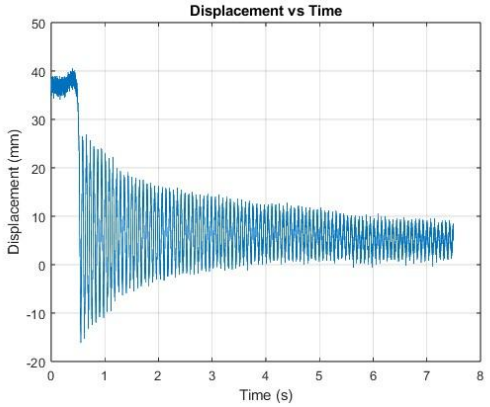


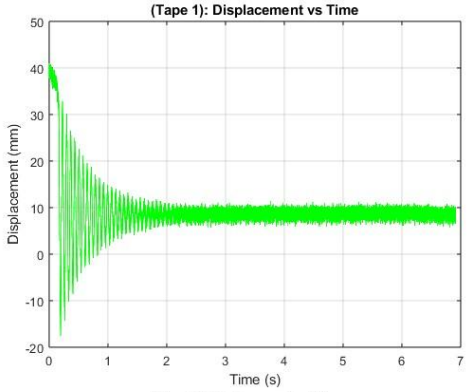
Figure #: Tapes Tested

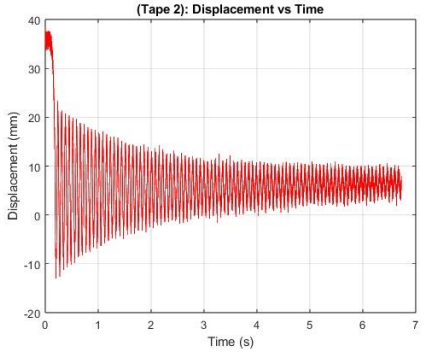
Vibration Control: Viscoelastic Damping

Case	Damping ratio (%)
No tape	0.25
Tape 1	0.56
Tape 2	0.4

- As can be seen in the graphs to the right cases 3 and 4 exhibited more damping than case 1 with case 3 having the most damping character. This is not reflected in the calculated zeta val







Dynamic Measurements: Summary

Parameters	Case 1	Case 2	Case 3	Case 4
1st Natural Freq. (Hz)	14.2303	14.1924	14.3598	14.1602
Damping Ratio (%)	0.2489	0.7091	0.5658	0.4045
Damped Nat. Freq. (Hz)	14.2292	14.1919	14.3595	14.1595

- As can be seen the cases 3 and 4 exhibited increased damping over case 1 and 2. This was expected

Conclusions

- Impact results in a drastically higher calculated zeta value despite a similar graphed response.
- Adding material will decrease the ability for the beam to vibrate via increasing its damping
- Impact results in a not purely case 1 vibration with somewhat significant secondary peaks

References

- [1] ME4005 Course Materials. Available: https://canvas.vt.edu/courses/187828
- [2] Omega Engineering, "Strain Gage Measurement," Omega Technical Reference, [Online]. Available: https://www.omega.co.uk/techref/pdf/straingage_measurement.pdf. [Accessed: Mar. 18, 2026].
- [3] H. Pendar, "ME 3524: Mechanical Vibrations," class lecture, Dept. Mechanical Engineering, Virginia Tech, Blacksburg, VA, Fall 2025.