Multibody Dynamics and Terramechanics-based modeling and simulation of Quarter-Car with Suspension [1]

Team 119

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Introduction

Problem

Existing quarter-car suspension models fail to account for the effects of terrain parameters, especially for deformable terrain, leading to unrealistically stiff off-road conditions in simulations.

Motivation

Integrating terramechanics with Tangent Space Ordinary Differential Equations (ODE) multibody formulations is essential to produce an accurate model of off-road terrain on a suspension equipped vehicle.

Methods

The research was done using simulation(s) not real-life tools and resources.

- **Tangent Space ODE Formulation**
  - Huag proves that Tangent Space ODE's are preferable to the traditional DAE methodology for modelling the non-linear dynamic system. [2][3][4]
  - The benefits of the Tangent Space method are most apparent in the increase in accuracy in modelling dynamic parameters
  - The root of the benefits comes from greater control on error and the simultaneous inclusion of several constraints

$$M(q)\ddot{q} - S(q,\dot{q}) + \Phi_q^T(q,t)\lambda - Q^A(q,\dot{q},t) = 0$$

$$D^T M D\dot{v} = D^T\left(MUB\gamma + S + Q^A\right)$$

$$\gamma = -\left(\Phi_q(q)\dot{q}\right)_q\dot{q} - \Phi_{tt}$$
$$U = \Phi(q_0)_q^T$$
$$B = \Phi(q)_q U^{-1}$$
$$D = I - UB\Phi(q)_q V$$

Figure 1: *The equation shown is the result of the tangent space method.*

- **Terramechanics Models**
  - Equations 7-9 are the Bekker-Wong normal stress relationships
  - Equations 10-13 are the Janosi-Hanamoto shear stress relationships
  - Shear stress is a function of normal stress

$$z = h(\theta) \tag{7}$$

$$h(\theta) = \begin{cases} r(cos\theta - cos\theta_f, \theta_m \le \theta \le \theta_f \\ r(cos\theta_e - cos\theta_f, \theta_r \le \theta \le \theta_m \end{cases} \tag{8}$$

$$\sigma(\theta) = \left(\frac{k_c}{b} + k_\phi\right) h(\theta)^n \tag{9}$$

$$\tau(\theta) = \left(c + \sigma(\theta)tan\phi_f\right)\left(1 - e^{\frac{-j(\theta)}{K}}\right) \tag{10}$$

$$j(\theta) = r\left(\theta_f - \theta - (1-i)\left(sin\theta_f - sin\theta\right)\right) \tag{11}$$

$$i = \frac{\omega r - V_x}{\omega r} \tag{12}$$

$$F_z = rb\int_{\theta_r}^{\theta_f}(\tau(\theta)sin + \sigma(\theta)cos\theta)d\theta \tag{13}$$

Figure 2: *The figure above shows the terramechanics equations.*

- **Parameters**
  - The (terrain) parameters are defined for sandy loam terrain

Parameter	Value
c	1700 Pa
k_c	5300
k_ϕ	1515000
ϕ_f	29°
ω	3 $rad\ s^{-1}$
V_x	0.35 ms^{-1}
b	0.075 m
r	0.25 m

Figure 3*: Terrain Parameters*

Parameter	*Value*
M	200 kg
m	71 kg
K	27600 Nm^{-1}
C	1000 Nsm^{-1}
mass of actuator plate	0.001 kg

Figure 4*: Linear Suspension Parameters*

Parameter	Value
M	200 Kg
m	20 Kg
K	37600 N/m
C	1000Ns/m
mass of actuator plate	0.001 kg
mass of body 2	3 Kg
mass of body 4	5 Kg

Figure 5*: Non- Linear Suspension Parameters*

- **Suspension Models**
  - Both are comprised of a sprung and unsprung mass. The unsprung mass is the wheel and its surrounding structure. The sprung mass is the rest of the model.
  - The models are quarter car models

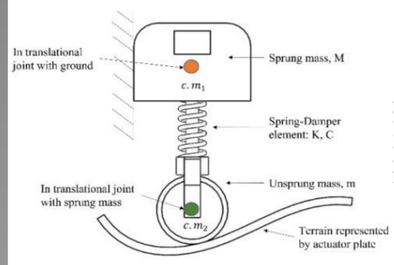


Figure 6*: Linear suspension model*

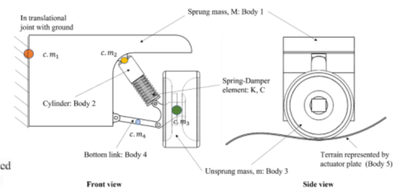


Figure 7*: Non-Linear suspension model*

Results & Discussion

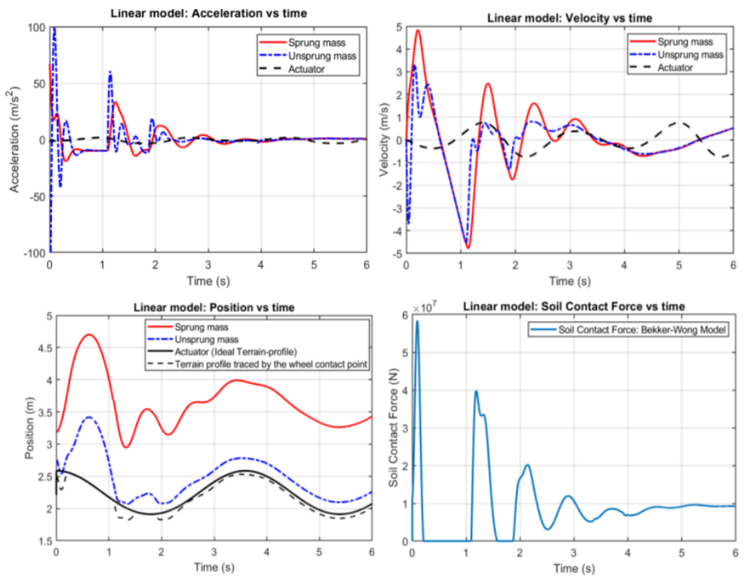


Figure 8: *Linear suspension model results*

- For the linear model, there is a relatively low amount of sinkage from the wheels in the sandy loam terrain. All parameters also reach steady-state behaviour after about 3 seconds.
- The transient response present high peaks of acceleration, velocity and force acting through the wheels.
- The soil contact force acting through the wheels is directly proportional to the sinkage raised to the n as expected by the Bekker-Wong relationship.

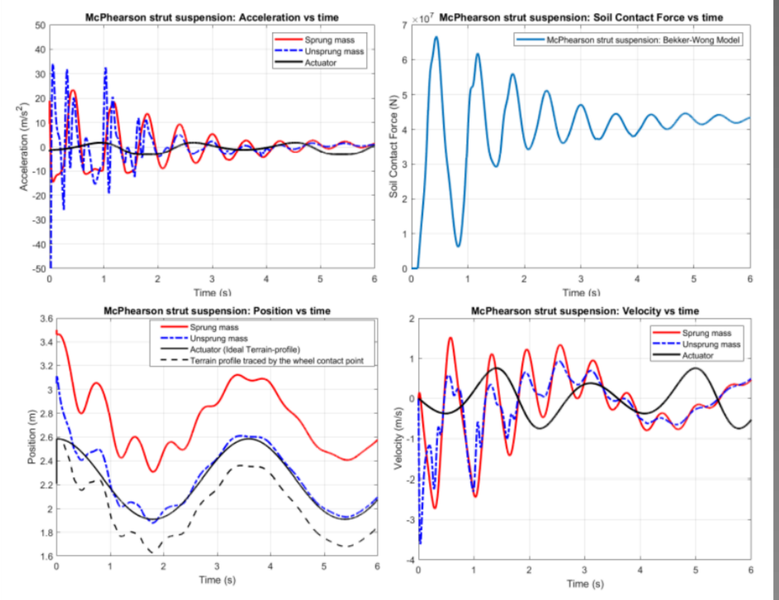


Figure 9: *McPherson suspension model results*

- The non-linear model (McPherson Strut) produces a high steady-state force through the wheels, and the sinkage is also higher than the linear model. The non-linear model reaches steady-state behaviour for all parameters after about 4 seconds.
- The McPherson Strut Suspension looks to be producing more vibrations and sinkage than the linear model and reaches steady-state behaviour at a later stage.
- Both models show that the application of tangent space ODEs in combination with terramechanics models produce viable results.
- The results of the paper suggest that multibody systems modelled with tangent space ODE's and Terramechanics equations apply to real life conditions.

Conclusion

- The research involved two distinct suspension types fitted to a quarter-car vehicle model.
- These models were subjected to a sandy loam deformable terrain simulation.
- The model and plots were generated using tangent space Ordinary Differential Equations (ODEs) combined with Bekker-Wong and Janosi-Hanamoto terramechanics forces.
- This suggests that terramechanics forces can be integrated with multibody dynamics models.
- Recommended future work includes:
  - Validating the simulation results using a real-world test model.
  - Working towards directly incorporating a neural network surrogate into the results.

References

[1] E. Chaturvedi, J. Mukherjee, and C. Sandu, "Multibody dynamics and Terramechanics-based modeling and simulation of quarter-car with suspension," IOP Conference Series: Materials Science and Engineering, vol. 1311, no. 1, p. 012020, Sep. 2024. doi:10.1088/1757-899x/1311/1/012020

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[4] He R, Sandu C and Osorio J E 2019 Systematic Tests for Study of Tire Tractive Performance on Soft Soil: Part I – Experimental Data Collection J. Terramech. 85 59-76